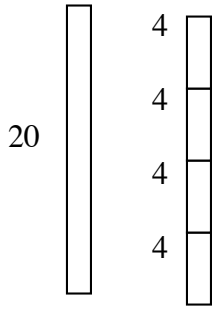
MAKING SENSE OF THE "INVERT AND MULTIPLY" ALGORITHM FOR DIVIDING FRACTIONS

Perhaps you know that when you divide fractions you can invert and multiply.
For example,

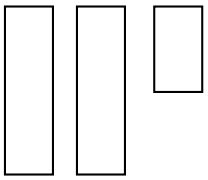
$$2\frac{1}{2} \div 1\frac{1}{4}$$
$$= \frac{5}{2} \div \frac{5}{4}$$
$$= \frac{5}{2} \times \frac{4}{5}$$
$$= \frac{20}{10}$$
$$= 2$$

But why does this work?
Remember that when we divide we can think of it as seeing how many times the divisor fits into the dividend. For example, when we divide 20 by 4 we want to know how many times 4 fits into 20.

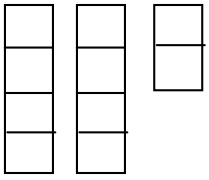


We can use the same approach with fractions.
For example: $2\frac{1}{2} \div \frac{3}{4}$

First we will represent the $2\frac{1}{2}$



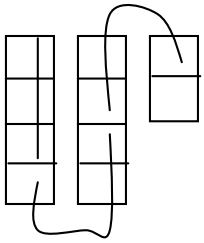
Then we will represent $2\frac{1}{2}$ in quarters because we know that we will want to see how many times 3/4 will fit into it.



Then we try to determine how many time 3/4 will fit into it.

It goes in 3 times and then a third of another time.

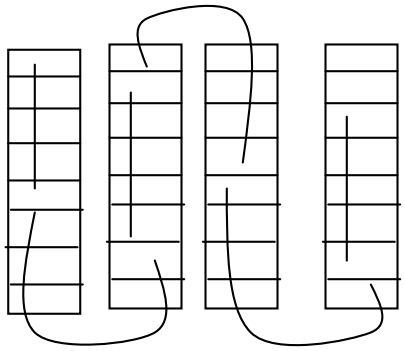
So the answer must be three and a third, or $3\frac{1}{3}$



If you try it, you'll see that this is the same answer we get with the invert and multiply method.

The reason it works is this: The algorithm tells us to invert and then to multiply. When you multiply something by a fraction, you multiply by the fraction's numerator and divide by its denominator. For example, 4x1/3 = (4x1)÷3. When you invert and multiply, you invert the fraction and so you end up multiplying by the second fraction's original denominator and dividing by its numerator. Multiplying the first fraction by the original denominator of the second will tell you how many pieces there are when the first fraction is split into pieces that are all the size of the second fraction's original denominator. Then you divide by the second fraction's original numerator. This tells you how many times that group will fit into the number of pieces you have. Here is one more example: 4 ÷ 5/8 = ?

The figure represents 4 divided into eighths. Then we see how many times a group of five eights can be taken from the four.



You can see that you get 6 whole sets of five eighths and then you can get two of the needed 5 additional eighths. This means that the answer should be 6 and 2/5.

Try drawing the diagrams for these two examples:

i. $3 \div \frac{3}{4}$ (Answer = 4)

ii. $2 \div \frac{3}{8}$ (Answer = $5\frac{1}{3}$)